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Increase in linear motion steadiness of tractor-trailers using stabilizing towing couplers

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Abstract. Travelling at higher speeds over any type of roads could disturb the stability of motion, which is manifested through lateral oscillations of the trailer in a horizontal plane – trailer wobbling. This research is aimed at building a mathematical model of tractor-trailer motion affected by the external action of lateral forces and estimating how efficient stabilizing couplers could be. A «double pendulum» design diagram is used to build the model. It is shown that the stability of motion is impacted at most by the ratio of the inertia radius (relative to the vertical axis passing through the center of gravity) to the base of the trailer, which depends on the type of cargo, its distribution within the trailer body, the heterogeneity of the cargo material and so on. It is demonstrated that the critical velocity of a trailer would increase when stabilizers are used. Changing the design of the trailer should also positively affect the stability of a tractor-trailer due to faster and more efficient damping of lateral oscillations impacting its movement.

1. Introduction

There are stricter requirements set on truck- and tractor-trailers (or just tractor-trailers) in terms of their operation safety. Travelling at higher speeds over any type of roads could disturb the stability of motion, which is manifested in lateral oscillations of the trailer in a horizontal plane – trailer wobbling. This phenomenon negatively impacts the road safety in general. Lateral deviations from a linear course widen overall and, therefore, traffic lanes of a tractor-trailer, increase its risk of skidding and leaving the dedicated lane, complicates driving as a whole – all this threatens oncoming or passing vehicles [1, 4, 6] and could lead to accidents. Known studies [1, 2, 3] cite data which implies that lateral oscillations of trailers in a horizontal plane could occur when a truck travels in a strict line but at a speed exceeding 35–40 km/h.

Nowadays, modern trucks and energy-saturated wheeled tractors feature quite advanced speed characteristics (above 80 km/h), while trailers themselves have hardly been upgraded. Thus, the problem of increasing the motion steadiness of tractor-trailers is certainly quite topical.

One solution of that problem aimed at damping lateral oscillations of trailers lies in the direction of using stabilizing towing couplers which connect the parts of such an articulated vehicle – a truck to a front bogie of a two-axle trailer, the bogie to the trailer platform.

To settle the motion of the front wheels of trailers equipped with a bogie connected to the trailer frame via a slewing ring, literary and patent sources describe various types of electromechanical, hydraulic and other stabilizing devices which generate a stabilizing torque relative to the kingpin of the ring.

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This research is aimed at building a mathematical model of tractor-trailer motion affected by the external action of such forces as lateral ones and at estimating how efficient stabilizing towing couplers could be in terms of their positive effect on the stability of linear motion.

2. Methods

To analyze how well mechanisms designed to damp lateral oscillations perform their task of stabilizing the linear movement of a two-axle trailer, its motion should be considered as a three-degrees-of-freedom mechanical system under the following assumptions: the trailer and its truck feature a rigid and gapless coupling; no vertical oscillations are present; the movement at that point where a front bogie is coupled with the truck is linear; the damping mechanism is installed on a rotary support connecting the front wheeled bogie and trailer platform.

To build a mathematical model, a «double pendulum» design diagram is used (figure 1). In a formalized form, the front bogie is considered to be the first section of the pendulum, the trailer platform – the second one.

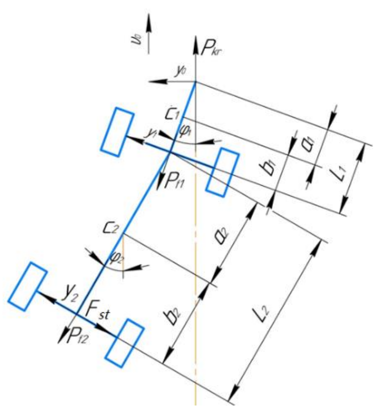

Figure 1. Diagram of two-axle trailer under horizontal lateral oscillations.

To describe the motion of this system, Lagrange differential equations are used.

The generalized coordinates are assumed to include: φ_1 – angle of deviation from a linear path of truck movement for the first trailed section; φ_2 – angle of deviation from the same path for the second trailed section; y – coordinate of the point of coupling on a truck-movement line.

The following external forces are applied to the system: P_{kr} – longitudinal component of hook power; y_0 – lateral reaction of the truck; y_1 – lateral reaction of the road applied to the front bogie tires; P_{f1} – rolling resistance of the bogie; y_2 – lateral reaction of the road applied to the trailer platform tires; P_{f2} – rolling resistance of the platform; F_{st} – stabilizing force due to a stabilizing device installed on the slewing ring, applied to the axis of the rear trailer wheels, thus helping them to return to the position of linear motion.

The kinetic energy of a system formed by two trailed sections equals to:

$$T=\frac{1}{2}(M_1+M_2)V_0^2+\frac{1}{2}\left(M_1a_1^2+M_2L_1^2+J_{C_1}\right)\dot{\varphi}_1^2+\left(M_1a_1+M_2L_1\right)V_0\dot{\varphi}_1\varphi_1+$$
$$+\frac{1}{2}\left(M_2a_2^2+J_{C_2}\right)\dot{\varphi}_2^2+M_2a_2V_0\dot{\varphi}_2\varphi_2+M_2L_1a_2\dot{\varphi}_1\dot{\varphi}_2.$$

where M_1, M_2 – masses of the front bogie and trailer platform, respectively, kg; J_{C_1} – inertia of the bogie relative to the vertical axis passing through its center of gravity, kg · m^2; J_{C_2} – inertia of the platform relative to the same axis, kg · m^2; V_0 – linear speed of the truck, m/s; a_1 – longitudinal coordinate of the center of gravity for the front wheeled bogie relative to the coupling loop of the truck, m; a_2 – longitudinal coordinate of the center of gravity for the trailer platform relative to the kingpin of the slewing ring, m; L_1 – base of the bogie, m.

Then, differential equations are formed:

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$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{y}}\right)-\frac{\partial T}{\partial y}=0,$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}_1}\right)-\frac{\partial T}{\partial \varphi_1}=\left(M_1a_1^2+M_2L_1^2+J_{C_1}\right)\ddot{\varphi}_1+M_2L_1a_2\ddot{\varphi}_2,$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\varphi}_2}\right)-\frac{\partial T}{\partial \varphi_2}=\left(M_2a_2^2+J_{C_2}\right)\ddot{\varphi}_2+M_2L_1a_2\ddot{\varphi}_1.$$

The generalized forces factor in the stabilizing force F_{st} and take the form of:

$$Q_y=P_{kr}-y_1\varphi_1-y_2\varphi_2-P_{f1}-P_{f2},$$

$$Q_{\varphi_1}=-y_1L_1-y_2L_1+P_{f2}L_1\left(\varphi_2-\varphi_1\right)+F_{st}L_1\left(\varphi_2-\varphi_1\right),$$

$$Q_{\varphi_2}=-y_2L_2-F_{st}L_2\left(\varphi_2-\varphi_1\right),$$

where L_2 – base of the second trailed section, m.

According to the studies of Zakin Y. H. [1], the lateral reactions of the road could be expressed as:

$$y_1=\frac{k_1L_1}{V_0}\dot{\varphi}_1+k_1\varphi_1,\qquad y_2=\frac{k_2L_1}{V_0}\dot{\varphi}_1+\frac{k_2L_2}{V_0}\dot{\varphi}_2+k_2\varphi_2,$$

where k_1, k_2 – coefficients of resistance to lateral drag for the tires of the first and second trailed sections, N/rad.

Since the first equation of the Lagrange system determines how large the longitudinal component of the hook power P_{kr} should be, a system of two differential equations is formed, which describes the motion of two trailed sections, that is a two-degrees-of-freedom system.

Considering that $J_{C_1}=M_1r_{C_1}^2$, $J_{C_2}=M_2r_{C_2}^2$, where r_{C_1}, r_{C_2} – radiuses of inertia for the first and second trailed sections relative to the vertical axes passing through their centers of gravity (m), a system of equations could be obtained:

$$\begin{cases}\left[M_1\left(a_1^2+r_{C_1}^2\right)+M_2L_1^2\right]\ddot{\varphi}_1+\dfrac{L_1^2\left(k_1+k_2\right)}{V_0}\dot{\varphi}_1+L_1\left(k_1+P_{f2}+F_{st}\right)\varphi_1+\\ \qquad\qquad +M_2L_1a_2\ddot{\varphi}_2+\dfrac{L_1L_2k_2}{V_0}\dot{\varphi}_2+L_1\left(k_2-P_{f2}-F_{st}\right)\varphi_2=0,\\ M_2L_1a_2\ddot{\varphi}_1+\dfrac{L_1L_2k_2}{V_0}\dot{\varphi}_1-L_2k_{st}\varphi_1+M_2\left(a_2^2+r_{C_2}^2\right)\ddot{\varphi}_2+\dfrac{L_2^2k_2}{V_0}\dot{\varphi}_2+L_2\left(k_2+F_{st}\right)\varphi_2=0.\end{cases}\tag{1}$$

A solution of the system (1) would be found in the form of $\varphi_1=d_1e^{lt}$, $\varphi_2=d_2e^{lt}$.

Substituting those expressions into (1), we get

$$\begin{cases}d_1\left\{\left[M_1\left(a_1^2+r_{C_1}^2\right)+M_2L_1^2\right]l^2+\dfrac{L_1^2\left(k_1+k_2\right)}{V_0}l+L_1\left(k_1+P_{f2}+F_{st}\right)\right\}+\\ \qquad\qquad +d_2\left\{M_2L_1a_2l^2+\dfrac{L_1L_2k_2}{V_0}l+L_1\left(k_2-P_{f2}-F_{st}\right)\right\}=0,\\ d_1\left\{M_2L_1a_2l^2+\dfrac{L_1L_2k_2}{V_0}l-L_2F_{st}\right\}+d_2\left\{M_2\left(a_2^2+r_{C_2}^2\right)l^2+\dfrac{L_2^2k_2}{V_0}l+L_2\left(k_2+F_{st}\right)\right\}=0.\end{cases}$$

The characteristic equation for this system is a quartic equation:

$$\alpha_0 l^4 + \alpha_1 l^3 + \alpha_2 l^2 + \alpha_3 l + \alpha_4 = 0.$$

Here

$$\alpha_0 = M_1 M_2 \left(a_1^2 + r_{C_1}^2\right)\left(a_2^2 + r_{C_2}^2\right) + M_2^2 L_1^2 r_{C_2}^2,$$

$$\alpha_1 = \frac{M_1 L_2^2 \left(a_1^2 + r_{C_1}^2\right) k_2 + M_2 L_1^2 \left(a_2^2 + r_{C_2}^2\right) k_1 + M_2 L_1^2 \left(b_2^2 + r_{C_2}^2\right) k_2}{V_0},$$

$$\alpha_2 = M_1 L_2 \left(a_1^2 + r_{C_1}^2\right)\left(k_2 + F_{st}\right) + M_2 L_1 \left(a_2^2 + r_{C_2}^2\right)\left(k_1 + P_{f2} + F_{st}\right) + \\ + M_2 L_1^2 a_2 \left(P_{f2} + 2F_{st}\right) + M_2 L_1^2 b_2 \left(k_2 + F_{st}\right) + M_2 L_1 L_2 a_2 F_{st} + \frac{L_1^2 L_2^2 k_1 k_2}{V_0^2},$$

$$\alpha_3 = \frac{L_1 L_2 \left[L_1 \left(k_1 k_2 + k_1 F_{st} + k_2 P_{f2} + 2k_2 F_{st}\right) + L_2 \left(k_1 k_2 + k_2 P_{f2} + 2k_2 F_{st}\right)\right]}{V_0},$$

$$\alpha_4 = L_1 L_2 \left(k_1 k_2 + k_1 F_{st} + k_2 P_{f2} + 2k_2 F_{st}\right),$$

where b_2 – distance from the rear axis of the second trailed section to its center of gravity, m.

Taking into account a low degree of the characteristic equation, Hurwitz stability criterion is used to check the motion stability of the system.

All coefficients of the characteristic equation remain positive under any practically conceivable changes of the parameters, which means:

$$\alpha_0 > 0, \quad \alpha_1 > 0, \quad \alpha_2 > 0, \quad \alpha_3 > 0, \quad \alpha_4 > 0.$$

Apart from having to satisfy these conditions, it is necessary to do the same for

$$\begin{vmatrix} \alpha_1 & \alpha_3 & 0 \\ \alpha_0 & \alpha_2 & \alpha_4 \\ 0 & \alpha_1 & \alpha_3 \end{vmatrix} = \alpha_1 \alpha_2 \alpha_3 - \alpha_1^2 \alpha_4 - \alpha_3^2 \alpha_0 > 0,$$

that is $\alpha_2 > \dfrac{\alpha_1 \alpha_4}{\alpha_3} + \dfrac{\alpha_3 \alpha_0}{\alpha_1}$, or, which is the same:

$$\frac{L_1^2 L_2^2 k_1 k_2}{V_0^2} > \frac{M_1 L_2^2 \left(a_1^2 + r_{C_1}^2\right) k_2 + M_2 L_1^2 \left(a_2^2 + r_{C_2}^2\right) k_1 + M_2 L_1^2 \left(b_2^2 + r_{C_2}^2\right) k_2}{L_1 \left(k_1 k_2 + k_1 F_{st} + k_2 P_{f2} + 2k_2 F_{st}\right) + L_2 \left(k_1 k_2 + k_2 P_{f2} + 2k_2 F_{st}\right)} \left(k_1 k_2 + k_1 F_{st} + k_2 P_{f2} + \right. \\ \left. + 2k_2 F_{st}\right) + \frac{L_1 L_2 \left[L_1 \left(k_1 k_2 + k_1 F_{st} + k_2 P_{f2} + 2k_2 F_{st}\right) + L_2 \left(k_1 k_2 + k_2 P_{f2} + 2k_2 F_{st}\right)\right]}{M_1 L_2^2 \left(a_1^2 + r_{C_1}^2\right) k_2 + M_2 L_1^2 \left(a_2^2 + r_{C_2}^2\right) k_1 + M_2 L_1^2 \left(b_2^2 + r_{C_2}^2\right) k_2} \times \\ \times \left[M_1 M_2 \left(a_1^2 + r_{C_1}^2\right)\left(a_2^2 + r_{C_2}^2\right) + M_2^2 L_1^2 r_{C_2}^2\right] - \left[M_1 L_2 \left(a_1^2 + r_{C_1}^2\right)\left(k_2 + F_{st}\right) + M_2 L_1 \left(a_2^2 + r_{C_2}^2\right) \times \right. \\ \left. \times \left(k_1 + P_{f2} + F_{st}\right) + M_2 L_1^2 a_2 \left(P_{f2} + 2F_{ст}\right) + M_2 L_1^2 b_2 \left(k_2 + F_{st}\right) + M_2 L_1 L_2 a_2 F_{ст}\right] = A_1 + B_1 - C_1.$$

If $A_1 + B_1 - C_1 \le 0,$ then this inequation is satisfied at any speed.

If $A_1 + B_1 - C_1 > 0,$ then

$$V_0^2 < \frac{L_1^2 L_2^2 k_1 k_2}{A_1 + B_1 - C_1} = \frac{L_2 k_2}{M_2 \frac{(A_1 + B_1 - C_1)}{M_2 L_1^2 L_2 k_1}}.$$

The conditions of stability by Hurwitz come down to a known expression found by Zakin Y H [1] which defines a critical velocity beyond which the stability of the system fails:

$$V_0 < V_{cr} = \left(\frac{L_2 k_2}{M_2 Z}\right)^{1/2},$$

where V_{cr} – critical velocity, m/s; $Z = A + B - C$.

But if a mechanism stabilizing lateral oscillations relative to the kingpin of the slewing ring were introduced into the system, the expressions for A, B, C summands would take the form of:

$$A = \frac{\left[\mu\left(\beta_1^2 + \chi_1^2\right) + \rho\left(\beta_2^2 + \chi_2^2\right) + \left(1 - \beta_2\right)^2 + \chi_2^2\right] \cdot \left[1 + \nu + (\rho + 2)\xi\right]}{\rho \cdot \left\{\gamma\left[1 + \nu + (\rho + 2)\xi\right] + 1 + \nu + 2\xi\right\}},$$

$$B = \frac{\left[\mu\left(\beta_1^2 + \chi_1^2\right)\left(\beta_2^2 + \chi_2^2\right) + \chi_2^2\right] \cdot \left\{\gamma\left[1 + \nu + (\rho + 2)\xi\right] + 1 + \nu + 2\xi\right\}}{\gamma \cdot \left[\mu\left(\beta_1^2 + \chi_1^2\right) + \rho\left(\beta_2^2 + \chi_2^2\right) + \left(1 - \beta_2\right)^2 + \chi_2^2\right]},$$

$$C = \frac{1}{\gamma\rho} \cdot \left[\mu\gamma\left(\beta_1^2 + \chi_1^2\right)(1 + \rho\xi) + \rho\left(\beta_2^2 + \chi_2^2\right)(1 + \nu + \xi) + \gamma\beta_2\rho(\nu + 2\xi) + \right.$$
$$\left. + \gamma(1 - \beta_2)(1 + \rho\xi) + \beta_2\rho\xi\right],$$

where

$$\mu = \frac{M_1}{M_2}, \gamma = \frac{L_1}{L_2}, \beta_1 = \frac{a_1}{L_1}, \beta_2 = \frac{a_2}{L_2}, \chi_1 = \frac{r_{C_1}}{L_1}, \chi_2 = \frac{r_{C_2}}{L_2}, \rho = \frac{k_1}{k_2}, \ \nu = \frac{P_{f2}}{k_1}, \ \xi = \frac{F_{st}}{k_1}.$$

Let us analyze how the stabilizing force F_{st} affects the critical speed on the example of a 2-PTS-4 tractor-trailer with a slewing ring and a stabilizing device installed on the ring. As a calculation basis, we take $M_2 = 6500$ kg, $L_2 = 3$ m, $a_2 = 1.5$ m, $k_1 = k_2 = 75000$ N/rad (the rigidity of front and rear wheels is identical), $f = 0.015$ (f – coefficient of rolling resistance), the mass of the bogie towbar could be neglected – $M_1 = 0$. Known research [4] shows that a two-axle trailer has sufficiently stable movement when its center of gravity is situated in the middle of its base. The stability of motion is affected at most by the ratio of the inertia radius (relative to the vertical axis passing through the center of gravity) to the base of the trailer, which depends on the type of cargo, its distribution within the trailer body, heterogeneity of the cargo material and so on. Consider the impact of that ratio on the critical speed without the stabilizing moment – $F_{st} = 0$. Figure 2(a) graphs calculated dependencies of the critical velocity V_{cr} on the ratio χ_2 of the inertia radius to the base L_2 of a two-axle trailer at various relations of L_1/L_2.

Analysis of the graphs in figure 2(a) demonstrates that the critical velocity diminishes for any correlation assumed between the base L_1 of the front bogie and the base L_2 of the two-axle trailer while the correlation χ_2 between the inertia radius to the vertical axis and the trailer base rises.

Consider the impact of the stabilizing force F_{st} on the critical velocity of a two-axle trailer at various χ_2 and for $\gamma = 0.7$. Figure 2(b) correlates the critical velocity V_{cr} and the stabilizing force F_{st} at various ratios χ_2 of the inertia radius to the trailer base.

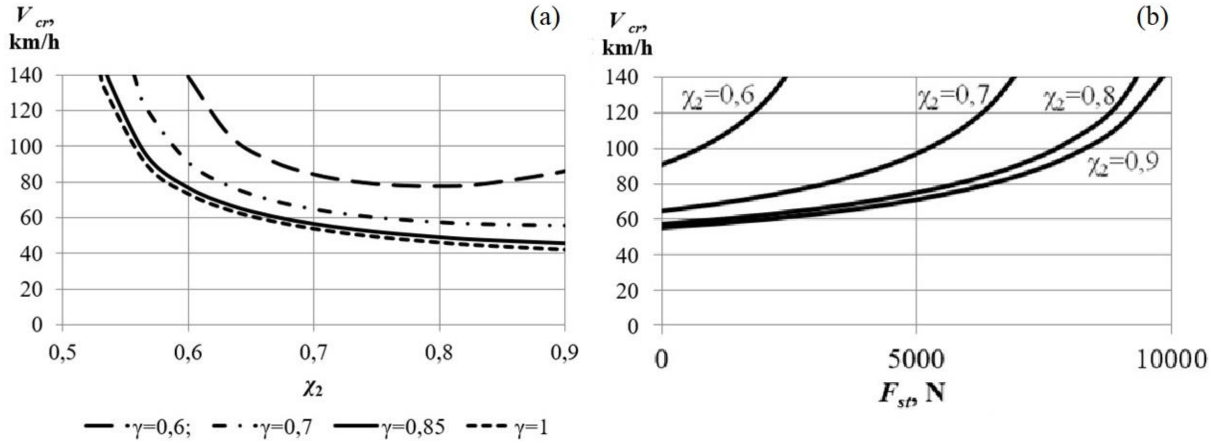

Figure 2. Dependency of the critical velocity V_{cr}: a) on the ratio χ_2; b) on the stabilizing force F_{st}.

3. Results and discussions

Analysis of the graphs shown in figures 2(a) and 2(b) allows to draw the following conclusions:

- The maximum impact on the stability and critical velocity of the trailer is caused by the ratio χ_2 of the inertia radius (relative to the vertical axis passing through the center of trailer gravity) to the trailer base. When this ratio rises and approaches 1, the results of calculation indicate that the critical velocity falls to 40 km/h. Such a velocity is too low for modern high-speed transport, and, at the same time, it depends on the type of cargo, its distribution within the trailer body and so on.
- Graphs which show how the critical velocity depends on the stabilizing force of a stabilizing rotary support installed on a trailer demonstrate that it would increase when stabilizers are used – the movement would become more stable.

4. Conclusion

Those conclusions were reflected in several patents on stabilizing towing couplers published by the authors [8, 9] and checked to be correct during laboratory tests carried out to study horizontal lateral deviations of a trailer on scale models of truck- and tractor-trailers by simulating their movement using a belt band at various speeds [5].

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